Chapter 9

Language Matters:

Mathematical learning and cognition in bilingual children

Mona Anchan and Firat Soylu

Abstract Although many bilingual children receive formal training in their non-dominant language in USA and other multicultural societies, educational programs tailored for the needs of bilingual children are scarce. Like in other areas of instruction, bilingual children face additional challenges when learning math, given the language divide between home numeracy and formal school environments. This chapter presents evidence-based recommendations for teaching math to bilingual and multilingual children in elementary and middle schools. To ground these recommendations in research findings, psychological and neural mechanisms of bilingual mathematical learning and cognition are discussed, as well as sociocultural issues and implications for classroom practice. To support bilingual children's math learning in their non-dominant language, we recommend allowing code-switching and other off-loading strategies, strengthening fact retrieval in both languages, incorporating the child's home and cultural contexts, instructing in their home language or finding online alternatives, providing culturally-relevant math instruction and feedback, and making connections between mathematics and children's everyday lives. We also discuss the need for changes in teacher training and educational policy-making, in order to increase awareness about bilingual children's needs and to transition bilingualism from being a disadvantage in formal education to being a quality that can enrich and enhance children's educational experiences.

Keywords Bilingual math, ELL, Math education, Math cognition, Bilingual children, Bilingualism, Home language, Teaching recommendations

9.1 Introduction

According to the 2019 national report about mathematical performance in USA, 16% of English-speaking children in 4th grade did not achieve basic profi-

M. Anchan (✉) • F. Soylu

The University of Alabama

Tuscaloosa, USA

e-mails: mona.anchan@gmail.com

fsoylu@ua.edu

ciency in math; this, however, is not as alarming as the 41% of 4th grade children categorized as bilinguals who lack basic mathematical proficiency (National Assessment of Educational Progress, 2019). With 23% of the U.S. school population being bilingual and growing (U. S. Census Bureau, 2021), understanding how bilingual individuals process mathematics is especially relevant for effectively teaching mathematics and improving their mathematical learning outcomes. This is especially important to ensure mathematics instruction is accessible and equitable for a subset of those bilingual children classified as English Language Learners (ELL) who constitute 10% of the U.S. school population (U. S. Department of Education, 2020).

If mathematics test outcomes were to serve as a proxy for children's understanding of basic mathematics (as they often do in policy circles), then the tradition of teaching mathematics in English or another dominant cultural language often yields difficulties for bilingual children. When bilingual children do not have access to educational opportunities or resources to learn basic mathematics in their home language early on, lack of connections between English and their home language may lead to academic difficulties in the short-term and potential long-term setbacks such as fewer career prospects and a lower quality of life (Dowker, 2019; Shin, 2013). However, these contextual matters are rarely considered in neuroscientific studies designed to understand how the brain processes different mathematical tasks. This unintentional oversight thereby continues to exacerbate existing inequities. For example, if most mathematical cognition studies that utilize neuroimaging proceed with the unspoken assumption that language proficiency does not play a primary role in mathematical processing by not classifying their participants' language status (see a list of all neuroimaging studies in numerical cognition in the review by Peters & De Smedt, 2018), then resulting recommendations to improve academic outcomes would incorrectly presume a person's language status as inconsequential for mathematics teaching and learning. Even considering the few mathematical cognition studies that do examine bilingualism, bilingual participants are treated categorically without qualifying their exact proficiency levels in their two languages.

Therefore, Whitford and Luk (2019) suggest treating bilingualism as the dynamic experience that it is. Most bilinguals, regardless of their proficiency, often have to perform mental gymnastics (Kroll et al., 2015) to cognitively manage the language system in which they are operating by activating the language they are currently using while suppressing competing representations in their other language. However, the quality and efficiency of bilinguals' mental gymnastics depend on their past and current interactions. For instance, a child who learns English as a second language in a natural/unstructured environment with high exposure to that language (e.g., home, relatives, neighbors, media, etc.) will have a different proficiency level than someone who learns English as a second language in a more structured environment like school (Bedore et al., 2016; Ruiz-Felter et al., 2016).

Since a child's language proficiency and bilingualism experience varies based on the interaction between their two languages and various environmental factors, Whitford and Luk (2019) suggest considering how bilingualism impacts cognition across ages and different sociocultural factors by examining the interaction between language factors (exposure, background, proficiency) and a variety of cognitive (e.g., verbal and non-verbal IQ) and demographic (e.g., education, socioeconomic status) variables. Understanding how bilingualism affects experiences (of mathematical learning or otherwise) is especially important now, given the economic and social shifts towards operating in multiple languages in this increasingly globalized world (Surrain & Luk, 2019). Outside of research that directly studies bilingualism, there is also a call to report and treat a child's language exposure, proficiency, and demographic variables in all developmental studies due to its potential to be a hidden moderator (Byers-Heinlein et al., 2019). Like the research sector, it is important to provide similar evidence-based recommendations for teaching and learning purposes as well. To provide such recommendations to mathematics educators who work with bilingual children and adults, findings from various fields (i.e., psycholinguistics, mathematics education, cognitive neuroscience, educational policy) were consolidated with insights from mathematical cognition about bilingual mathematics learning in international contexts.

While the cognitive demands of bilingualism are ever present in a bilingual child's development (Whitford & Luk, 2019), these cognitive demands seem to unequally affect students' mathematics performance (Anchan and Soylu, 2021d), which could have downstream effects on their future numerical development. Understanding numerical development requires connecting theories and findings across different levels, and studying how environmental and cultural factors (e.g., language, home numeracy environment, socio-economic level), as well as biological factors (e.g., neural, genetic) contribute to the development of numerical skills.

In this chapter, we utilize the interdisciplinary lenses of educational neuroscience to present how bilingual children in the elementary and middle school years develop their mathematical thinking and learn mathematics (Han, Soylu, & Anchan, 2019; Knox, 2016). The pragmatic epistemology of this framework allows an educational concern (i.e., poor mathematical performance in bilingual children) to define a course of action by drawing on interdisciplinary insights about children's language proficiency, their cognitive processes, and sociocultural factors related to their school and home environments. But extrapolating insights from multiple disciplines has its own challenges. For example, it is not always possible to keep terminology consistent while crossing disciplinary boundaries. Therefore, at the risk of compromising precision for a pragmatic cause, we have used the term 'bilingual' broadly in this chapter, only qualifying bilingual proficiency when reported by the original study. It is our hope that this lack of precision will advance research in the areas of bilingual mathematical cognition and mathematics education by prompting further conversations and discussions not just across academic disciplines but also among practitioners, administrators, and parents.

9.2 Biological and cultural evolution of mathematical skills

Given that human mathematical skills are to a large extent an outcome of recent cultural evolution, the human brain does not have dedicated systems that originally evolved to support mathematical cognition. Instead, similar to many other cognitive skills, mathematical cognition makes use of neural systems that evolved to support other functions (Anderson, 2010, 2014). Early studies focused on understanding the role of language, symbolic, visuospatial, and sensorimotor systems for fundamental mathematical skills, like numerosity estimation, subitizing, counting, and arithmetic (e.g., Dehaene, 1992). Across nearly 30 years of research, we learned that we share some fundamental mathematical skills with other animals, enabling estimation of physical and numerical magnitudes, but beyond that mathematical development strongly relies on body-based, visuospatial, symbolic, and verbal representations, which are embedded in sociocultural contexts where development takes place (O'Shaughnessy et al., 2021). But with the world becoming increasingly connected and globalized, evidence about bilingual brains is calling a fundamental assumption in these studies into question (Whitford & Luk, 2019). Can mathematical cognition findings about neural processing be extended to all populations when many of these studies did not account for individuals' language status (monolingual, bilingual, multilingual) or include it as a variable? And how does this affect how bilingual children are taught math? With this goal in mind, this chapter outlines evidence about brain development and mathematical processing in bilinguals, followed by some recommendations to integrate these findings in the mathematics classroom (or home) while teaching bilingual children.

9.3 Bilingual brains process information differently

While monolingual and bilingual brains both process languages and cognitive tasks efficiently, the brain networks carrying out similar tasks and the associated outcomes may differ (Anderson et al., 2018). The fundamental architecture and language processing mechanisms involved in the bilingual brain may not always be accessible or examinable in monolinguals (Kroll et al., 2015). This insight is reiterated in neuroimaging and behavioral studies that show monolingual and bilingual children and adults performing differently on similar tasks or using different brain networks (Bialystok, 1999; Bialystok & Martin, 2004; Anderson et al., 2018). For example, in an fMRI study, Anderson et al. (2018) compared monolinguals and English-French bilingual adults on verbal and nonverbal task-switching experiments. While monolinguals used 2 different networks to process the verbal and non-verbal tasks, bilinguals used a common network for both tasks. Other studies suggest that bilinguals have enhanced executive functioning skills such as attentional control (Bialystok & Majumder, 1998; Bialystok, 1999), mental flexibility (Mielecki et al., 2017) and inhibitory control (Bialystok & Martin, 2004; Kroll et al., 2008; van Heuven & Dijkstra, 2010) as a result of juggling two languages.

9.4 Bilingual mathematical development

The implicit assumption in many elementary and middle school classrooms is that mathematics is learned in a language-independent way (Anchan, 2019). A growing body of neurocognitive evidence suggests otherwise. When examining mathematical problem solving in Turkish-German bilingual elementary school students, for example, Kempert et al. (2011) found language proficiency in the language of instruction/testing to be predictive of their mathematical performance. Bilinguals activate mathematical representations in both languages at all times. Therefore, they would either actively inhibit representations in one language while performing operations in the other (Kroll et al. 2008) or they would use some cognitive subprocess to choose one language over the other (Dijkstra and Van Heuven, 2002). As a result of these background processes, bilinguals may react significantly slower than monolinguals on some tasks. Juggling additional subprocesses (Kroll, 2008) may also lead bilinguals to make more errors on mathematical tasks, leading to some disadvantages in bilinguals. Venkatraman et al. (2006) fMRI-scanned English-Chinese bilinguals as they performed two arithmetic tasks—base-7 addition and percentage estimation—to study exact and approximate number processing. They performed the tasks in both English and Chinese where they were trained in one language and untrained in the other. Language switching effects were found in both types of number processing – approximate number processing (left inferior frontal gyrus [LIFG], left inferior parietal lobule, angular gyrus), and exact number processing (bilateral posterior intraparietal sulcus, LIFG) – suggesting that mathematical calculation (which depends on retrieval of mathematical facts) relies on verbal and language-related networks. Therefore, mathematical processing is not independent of language.

More specifically, mathematical retrieval, calculation, and performance seem to depend upon the primary language of mathematics instruction. When bilingual high schoolers were trained on multiplication and subtraction problems in one language (German or French) and tested in both languages, Saalbach et al. (2013) found cognitive costs related to language switching when language of arithmetic instruction differed from the students' frequently used language. Similarly, in a sample of 193 German-French bilinguals between the ages of 12-23, Van Rinsveld et al. (2015, 2016) found bilingual participants' language proficiency to be crucial for solving simple and complex addition problems. While extended amounts of practice in both languages helped bilingual participants to perform equally well on simple single-digit addition problems, this was not true for more complex addition problems that involved double-digit or larger numbers. The number words used to describe the numbers also made a difference in how bilinguals processed numbers. For example, a bilingual whose primary language is English may read 24 as "twenty-four" but a bilingual who primarily speaks German may read is as "four-and-twenty." These small but significant differences seem to compound over time leading bilinguals to process mathematics problems faster and more easily in their first language. In the case of simple multiplication, Salillas and Wicha (2012) similarly showed that the memory networks established in a bilingual individual's childhood does not affect their retrieval process in adulthood even if the other language is dominant.

While many of the previously mentioned studies support the notion that arithmetic facts are encoded in verbal memory in the language of mathematics instruction (Dehaene & Cohen, 1995), there is also evidence to suggest that bilinguals represent mathematical facts for each language separately (Campbell & Xue, 2001). Martinez-Lincoln et al. (2015) compared bilingual teachers' arithmetic performance in their primary or secondary language of instruction. They found that when the teachers performed arithmetic in their primary language of instruction, they maintained a primary language advantage. When their performance in their teaching and non-teaching languages were compared using Event-Related potentials (ERPs, i.e., brain signals), teachers showed more efficient access to their language they taught in, regardless of whether it was their first or second language. This suggests that access to terms even in the secondary language of instruction improved with use and practice. Although this study was done in adults, it could have implications for mathematical learning in children.

Cerda et al. (2019) recorded ERPs in bilingual children as they verified the correctness of multiplication problems that were presented as spoken number words in Spanish and English blocks. Even though participants showed a language bias, they elicited comparable N400 amplitudes (i.e., brain response to encountering something unexpected that usually happens when the number presented does not match the expected answer) for both languages, which suggests similar cognitive processes in both registers at the semantic level. According to these adult and child studies, if a bilingual child's development in both languages is almost balanced, disadvantages arising from mathematics instruction in their second language could be mitigated by increased functional use of their second language. This, however, may not be the case for bilingual children with partial or limited use of their second language such as English Language Learners. For such bilinguals, Van Rinsveld et al. (2016) found that providing contextual cues in their home language during instruction helped bilingual participants perform better in mathematics even in their second language. This means that despite their level of proficiency in both languages, bilingual individuals must learn mathematical facts in both languages to retrieve them at the same rate; otherwise, they will face cognitive costs when performing in their second language, because they would most likely retrieve facts in the language of instruction and translate them into the other language (Schwartz and Sprouse, 1996).

9.5 Insights from bilingual mathematical education in USA and other countries

Research about bilingual mathematical education corroborates much of the cognitive findings about bilingual mathematical development presented earlier. For example, while examining the math learning success of Filipino-English bilingual children in the 5th grade, Bernardo and Calleja (2005) found that they were more likely to understand and solve word problems in their first language. This first language advantage was observed even in bilinguals whose first language was English (Bernardo, 2002). A parallel observation was documented outside the USA, where Clarkson and Galbraith (1992) found Papua New Guinean 6th grade students who were proficient in both their languages (English, and a second native language, e.g., Tok Pisin or Hiri Motu) scoring considerably higher on two different mathematical tests compared to their less proficient bilingual peers. Some bilingual students even performed better than their monolingual peers, despite the latter belonging to schools with more resources. Further examination led Clarkson (1992) to conclude that this may be due to Papua New Guinea's national policy promoting the use of students' original languages in school, allowing them to easily understand difficult mathematical concepts in classrooms. Planas and Civil (2013) compared students from Mexico in Tucson, USA, and students from

Latin America in Barcelona, Spain. In both cases, the primary language of instruction were English and Catalan respectively. They showed that students' level of participation in the mathematics classroom depended on the language of instruction; low levels of participation were associated with instructing in students' non-home language. These studies from other countries support the recommendation to incorporate bilinguals' home language in mathematics instruction in U.S. classrooms as well.

9.5.1 Frequency of language use

Mathematical cognition models in sync with insights about word frequency (Ashcraft, 1992) and information processing (Anderson, 1983) state that when a language in which mathematics is taught and learned is used frequently, those mathematical facts will be stored and retrieved in that language most efficiently. Campbell and Clark's (1988) encoding-complex model of mathematical cognition also posits that each language has its own representation of arithmetic facts, and the rate of retrieving those facts is dependent on experience (Campbell & Epp, 2004). Since 'reaction time' or the time to access mathematical facts and solve problems is a primary measure of mathematical performance in schools, understanding how efficiently children retrieve their learned mathematical facts (which depends on experience and practice) is important for teaching mathematics to bilingual children. The frequency of language use seems to be important to children (Thordardottir, 2019) not only when they are learning mathematics but also while maintaining learned facts through retrieval-based procedures such as practice and problem-solving. In the next section, based on a review of bilingual and mathematical education research in the USA as well as other countries, we make evidence-based recommendations for math teachers and parents to reduce elementary and middle school bilingual children's cognitive load, supplement their math learning processes, and improve their math learning outcomes.

9.6 Evidence-based Recommendations

9.6.1 Allowing code-switching

An overwhelming body of research seems to suggest that 'code-switching' or switching between their two languages should not be discouraged or penalized among bilingual children, thereby encouraging, and even normalizing bilingual instruction for all children. Parvanehnezhad and Clarkson (2008) studied language switching in Iranian bilingual children as they solved mathematical problems and explained their reasoning in an interview setting. Students reported switching between their home language (Farsi) and the language of instruction (English) when they found the problem to be difficult, when they were more familiar with the Farsi version of the numbers or words being used, and when they were in a Persian school environment. It is equally important to consider the cognitive processes a bilingual student may be employing while learning mathematics in either language so instruction and communication of mathematical concepts can be tailored to their 'zone of proximal development' (Zaretskii, 2009). In addition to promoting students' understanding of mathematical concepts, teaching mathematics in students' first language also seems to nurture socioemotional aspects of their learning. In a comparison of five multicultural schools in Sweden, bilingual students between the ages of 9 and 16 reported higher levels of confidence, engagement and learning when bilingual mathematics teachers instructed their students and engaged them in mathematical activities using both languages. Students also felt secure using both languages while doing and understanding mathematics problems (Norén, 2008).

9.6.2 Allowing other 'off-loading' strategies

Similar to code-switching, implementing other sensorimotor strategies (e.g., finger counting, sketching, diagramming, visual aids, etc.) can also help bilingual children to offload some of their persistent cognitive load, thereby allowing for better mathematical processing and performance. Children who use their fingers to count and do arithmetic in the early school years (K to 2nd grades) were found to perform better in the later school years (Baroody & Wilkins, 1999; Crollen & Noël, 2015; Long et al., 2016). Before children switch to mental number representations entirely, fingers help children as a cognitive offload or embodied processing mechanism, later to be replaced by fact retrieval, which is more efficient and makes cognitive resources available for learning of more advanced arithmetic and algebra. Neuroimaging studies both with children (Berteletti & Booth, 2015) and adults (Soylu & Newman, 2016) show an association of the finger sensorimotor system with number processes. In addition, multiple studies showed that children's finger gnosis (the ability to individuate fingers) scores correlate with or predict their mathematical skills (Fayol et al., 1998; Noël, 2005), even though there are also some studies not showing such an association (Long et al., 2016). There are also studies showing that fine motor ability correlates with (Fischer et al., 2017) or predicts (Luo et al., 2007) mathematical skills in young children. Similarly, visual mathematical representations (VMRs) and computer-based Mathematical Cognitive Tools (CMTs) help educators to scaffold their instruction and help children to cognitively offload while learning (Sedig & Liang, 2006).

9.6.3 Strengthen retrieval of mathematical facts in both languages

Since basic mathematical facts are the building blocks of higher-level math, strengthening bilingual children's retrieval of basic mathematical facts in both their languages is crucial for their mathematical development. Neuroimaging studies show that with higher arithmetical skills, both children (Rosenberg-Lee et al., 2011) and adults (Grabner et al., 2007; Prado et al., 2011) show less activation in the intraparietal sulcus (i.e., associated with calculation) and more in angular gyrus (associate with retrieval) during arithmetic tasks, particularly for addition and multiplication, given the higher reliance on retrieval of arithmetic facts for these operations. Further, in a study conducted with adults, parietal activation during a complex multiplication task shifted from intraparietal sulcus (i.e., calculation) to angular gyrus (i.e., retrieval), as these adults were trained with the multiplication facts included in the task (Grabner et al., 2009). Automatizing calculation processes by switching to retrieval via practice could be another way to help students reduce their cognitive load.

9.6.4 Incorporating home and cultural contexts

Another recommendation to improve bilingual children’s learning outcomes in mathematics is to draw on the children’s home and cultural contexts. Whether monolingual, bilingual, or multilingual, Secada and De La Cruz (1996) suggested that students need to make sense of mathematics instruction in order to perform satisfactorily, and that is often achieved by connecting children's problem-solving strategies to mathematics instruction in school. Building on this suggestion, they recommend using children's home and cultural backgrounds to promote mathematical understanding among students from varied cultural and linguistic backgrounds. According to Secada and De La Cruz (1996), teachers should consider adopting these four principles in their teaching practice: (1) assessing students’ understanding constantly; (2) allowing students to choose from a variety of mathematical content and levels that is interesting, open-ended, and accessible; (3) building on students' prior knowledge and home experiences; (4) developing mathematical language in their cultural and linguistic context. For bilingual students, the last two suggestions are all the more crucial from a standpoint of equity; it would allow them to build their own understanding of mathematical concepts from a similar starting point as their monolingual peers.

9.6.5 Mathematics instruction in the home language

While it is recommended that bilingual children’s home language is utilized for mathematics instruction, the context, setting, and manner of this instruction should also be considered. For example, the Redwood City study for Mexican American bilingual children (Cohen, 1976) found that separating bilingual children from their monolingual peers to instruct them in Spanish did not necessarily have the desired effect in academic achievement. In a study of 3rd to 4th grade children who were instructed bilingually for six years, bilingual children outperformed their public-school peers (instructed in English) in Spanish reading, vocabulary, and storytelling. However, their public-school peers, whether monolingual or bilingual, performed better in English storytelling, which means that separating bilingual students could result in them having lesser opportunities to practice syntactic improvisation in English. The comparisons between the bilingual-schooled and public-schooled children yielded mixed results in mathematics and English vocabulary performance.

A meta-analysis of bilingual education programs conducted by Willig (2012) found similar mixed results with small to moderate differences favoring bilingual education in reading, language skills, mathematics, and total achievement when the tests were in English, and in reading, language, mathematics, writing, social studies, listening comprehension, and attitudes toward school or self when tests were in other languages. The mixed performance in mathematics and other subjects seen in these studies further suggest that separate instruction may not be the most effective strategy in mathematics instruction for bilinguals.

Since most classrooms have bilingual children who speak more than one non-English language which mathematics teachers may not speak themselves, it is not practically feasible to instruct all bilingual children in their home language. One recent innovation that some mathematics teachers have implemented to deal with this challenge is to connect with teachers in other countries teaching the same material and provide students some 1-on-1 online instruction or recordings of the instruction in the student's home language (WestEd, 2020). Teachers also reported organizing their classes in rotating stations where students with various home languages were grouped together and each station included instruction in a different home language; students would then take turns learning from the instruction at each station, wherein the student familiar with the language of instruction would explain what they learned to their station peers who were not familiar with the language, thereby 'flipping the script' (WestEd, 2020). Despite such creative solutions, there is a shortage of mathematics instructional practices that is inclusive for all bilingual students.

9.6.6 Immersive bilingual programs or Structured Immersive sessions

In 1996, Rossel and Baker reported only 25% of the 300 evaluated immersive bilingual programs to be methodologically acceptable. Surprisingly, Willing (2012) reported similar results 16 years later. Willig (2012) called for quality research in the field of bilingual education to remedy the high prevalence of methodological shortfalls seen in this domain. This suggests that there has been a need for effective bilingual programs in USA for at least 25 years. According to Rossell and Baker (1996), only 9% of the methodologically acceptable programs showed bilingual education to be more effective than regular classroom instruction for mathematical performance.

A few studies found bilingual education more effective than regular classroom instruction but even then, structured immersion was still considered as the ideal format for bilinguals with limited English proficiency. Structured immersion programs help bilingual children acquire language skills in their second language so they can succeed in a classroom where mathematics and other instruction occurs primarily in English. A meta-analysis conducted by Greene (1997) similarly recommended using their native language (versus English-only instruction) when instructing bilingual children with limited English proficiency for moderate learning benefits.

From a teaching point of view, therefore, it seems that in addition to some educational innovation, providing mathematics instruction in a bilingual child's home language in early elementary grades without separating them from their monolingual or balanced bilingual/ multilingual peers might prove most beneficial. This could be done in the form of supplementary structured immersion sessions where English is taught explicitly to low-proficiency bilinguals who are grouped and instructed according to their English proficiency. Another way to do this might be to start or maintain quality two-way immersion (TWI) programs that have succeeded in preparing bilingual students for better mathematical understanding and performance in both their languages. Lindholm-Leary and Borsato (2005) examined general school-related attitudes, mathematics coursework, and mathematical achievement in three groups of high school students—Hispanics who used to be ELLs, native English-speaking Hispanics, and Caucasian English-speaking monolinguals —enrolled in a TWI program throughout all the elementary grades. They found all three groups had positive attitudes toward mathematics and school, were enrolled in college preparation mathematics courses, and were performing at average or above average levels in math. Marian et al. (2013) confirmed this finding in their comparison of test scores across different elementary school programs. They found that Bilingual TWI programs positively affected students' mathematics and reading performance regardless of their language status. Bilingual students in TWI programs outperformed their peers in transitional programs of instruction. Similarly, English-speaking students in TWI programs outperformed their peers in regular classrooms.

9.6.7 Feedback and culturally relevant mathematics instruction

But any program or type of instruction is only as effective as the sum of its components, and educators play the most vital role in this equation. This is very much in line with Clarkson and Gabraith (1992) who cautioned against treating bilingualism as a unidimensional factor, and instead advocated designing research and programs by accounting for the myriad of factors that play a role in educating bilingual children. Teacher-driven learning supplements, such as feedback and culturally relevant mathematics instruction, is one such factor that has been shown to be effective in instructing bilingual children. Cardelle-Elawar (1990) trained four pre-service mathematics teachers to provide oral feedback to their low-performing 6^{th} graders who were bilingual. The oral feedback was modeled on Mayer's model of metacognition and 4-step-problem solving: (1) translation, (2) integration, (3) planning and monitoring, (4) solution execution. Effective implementation of this model showed that just 6 hours of feedback led to higher mathematics performance in low-performing bilingual students.

Cahnmann and Remillard (2002) qualitatively examined teachers' role in making mathematics instruction accessible to students from diverse backgrounds. Individual cases found two effective strategies for instructing bilingual students in math, given the implicit assumption that they are teacher-generated: (1) drawing connections between the student's culture and mathematical concepts; (2) pursuing the complexities of mathematics and making it meaningful to the student. Cahnmann and Remillard also called on administrators to provide generous scaffolding and support to their teachers so they, in turn, can provide similar levels of support to their bilingual students.

9.6.8 Discussions about mathematics and culture

Supporting teachers as they do this complex non-formulaic work of teaching and supporting bilingual students is crucial due to the dearth of existing structural supports in the educational system. Bose and Remillard (2011) examined national policy reports detailing U.S. mathematics instruction to identify ways to render mathematics education more equitable. Due to its definition and focus being restricted to procedural and factual knowledge, resulting recommendations for mathematics instruction focused on supporting teacher content knowledge over other forms of knowledge. This is unfortunate since evidence suggests that teachers must wield various types of knowledge and skills to teach students effectively and further facilitate student learning.

For example, Bernardo and Calleja (2005) showed that bilingual students usually neglected to consider real-life constraints and connections while solving word problems. This can be easily rectified if teachers are supported and encouraged to teach mathematics in the context of a student's everyday experiences (including language) instead of the typical procedural manner devoid of linguistic markers. Moschkovich (2007a, 2007b) suggested that having a mathematical discussion with bilingual students would draw on existing sociolinguistic resources allowing them to make meaning of mathematical concepts and integrate it into their lives more willingly. Dominguez (2011) similarly advocates capitalizing on students' experiences as bilinguals as cognitive resources to teach math. In their study, pairs of students who solved problems showed differences in their communication and thought patterns, depending on the context and language.

9.6.9 Making connections between mathematics and aspects of children's lives

There are additional strategies used by elementary and middle school teachers in regular classrooms that, not so surprisingly, have been found to be effective in instructing bilinguals. Gutiérrez (2002) highlighted three high school mathematics teachers who successfully instructed many Hispanic students. The strategies they used to do so included building on students' previous knowledge, using supplementary textbook materials, and promoting teamwork which allowed students to work in their primary language alongside peers. Musanti et al. (2009) conducted a case study of a first-grade bilingual teacher learning and teaching Cognitively Guided Instruction, a framework used to understand student's understanding of contextualized word-problems. They found that ongoing reflections, collegial conversations, and constantly analyzing students' work enriched a teacher's understanding of how students learned math; this, in turn, allowed them to provide more opportunities for students to explain their solutions and thinking, thereby creating an effectual feedback loop between instruction and performance. Therefore, instructing bilinguals in their home language or a mixture of both languages while connecting the content to their own life and culture in the form of discussions, reflections, and teamwork might foster understanding of mathematical concepts and boost mathematical performance in bilinguals.

9.6.10 Confirmations from non-USA contexts

The recommendations suggested in previous sections have also been replicated in countries outside North America. Gale et al. (1981) tested elementary school children's academic performance in both English-only and bilingual program classes in Milingimbi, an Aboriginal community in Australia. Although not immediately apparent, by their seventh year in the program, the children enrolled in bilingual classes outperformed their English-only peers in seven out of ten tests, mathematics being one of them. Based on a study of bilingual students in Norway, Özerk (1996) similarly suggested that a case could be made for the adoption of bilingual education grounded in pedagogical evidence. Özerk compared mathematics teaching and learning between two groups of linguistic-minority Norwegian students; one group was instructed in the students' second language (typical monolingual setting like most U.S. classrooms) and the other in a bilingual classroom. The linguistic-minority students instructed in a bilingual classroom performed at the same level or better in mathematics than their peers from the monolingual classroom. Based on high attendance and promotion rates coupled with low dropout rates for Guatemalan bilingual schools, Patrinos and Velez (2009) proposed switching from regular to bilingual education programs for students belonging to disadvantaged populations, estimating national cost savings of $5 million. According to them, students enrolled in bilingual schools performed above average on all subject matters.

These cross-sectional findings were further substantiated by a 4-year longitudinal study in The Netherlands investigating the effects of using English as the language of instruction during the first four years of secondary education (Admiraal et al., 2006). Academic performance of these bilingual students, who were equally proficient in English and Dutch, was compared to students instructed in Dutch. They found that the students enrolled in the bilingual program outperformed their peers in regular classrooms, and also showed higher English language proficiency. The same was found to be true in Cambodia (Lee et al., 2015) and Mozambique (Benson, 2000). Lee et al. (2015) recommended using a student's first language for mathematics instruction to foster understanding of mathematical concepts, much like researchers' recommendations in the U.S. context. Similar to the U.S. context, Benson (2000) found teaching in two different Bantu languages in transitional bilingual programs more promising for educating bilinguals than instructing them in their non-native language. While transitional programs are a step in the right direction when considering equitable bilingual mathematics education, international data lends support to U.S. findings about two-way immersion programs suggesting that they still might be the most effective for instructing and educating bilingual as well as monolingual children.

Similar support for TWI programs was found in a Canadian study. Math, English, and French performance of elementary-school students (ages 6-12) in Montreal's four different public-school programs—French-as-a-second-language, delayed and early French immersion, and full French-medium schooling (i.e., teaching all subjects in French)—were compared in a longitudinal study (Lambert et al., 1993). The control group for this study consisted of students enrolled in an all-English and all-French school. Except for French oral skills, they found students in French-medium and French immersion programs to be indistinguishable from students in all-French schools on written aspects of French, English, and Math. This also supports the view that students' oral proficiency in a language is determined by the opportunities for social interaction available to them.

9.6.11 Need for more innovation and research

Although American and international research corroborates the benefits of bilingual education and the importance of teaching mathematics in a child's home language, its effectiveness is contingent upon thoughtful implementation of bilingual instruction as well as consideration of students' needs. Examining the implementation of mathematics instruction in Malaysian bilingual classrooms, Lim and Chew (2007) pointed out that approaching mathematics instruction in a procedural manner when instructing in a non-dominant language led to poor understanding of mathematical concepts among students and therefore, poor mathematical performance. In the same vein, Tsung and Cruickshank (2009) showed the detrimental effects on mathematical performance when students do not receive adequate instruction in their mother tongue. They conducted case studies of two schools in China, a rural minority elementary school that instructed in a minority language (not Mandarin or Cantonese) and an urban mixed minority elementary school where Mandarin was the primary language of instruction. Minority ethnic children performed poorly in all three—their mother tongue, Mandarin, and English—compared to their peers instructed in Mandarin. These studies suggest that teaching bilingual children mathematics in their home language in an immersive environment is necessary but not sufficient. To address the need for more research and innovation, educators and parents are encouraged to partner with interdisciplinary researchers (e.g., educational psychologists/neuroscientists) so their practical knowledge and insights can become an integral part of the efforts to improve math teaching and learning for bilingual and multilingual children (Anchan, 2022).

9.7 Conclusion

Using an interdisciplinary lens, this chapter outlined the similarities and differences in how bilingual children learn and process mathematics compared to the established monolingual norm. To avoid conceptualizing bilinguals as two monolinguals in one (Grosjean, 1989), recommendations were made for teaching mathematics to bilingual children in the elementary and middle school years based on studies from the fields of mathematics education, educational policy, and educational psychology which were further supported by evidence from cognitive neuroscience, mathematical cognition, psycholinguistics, and cognitive science. Proper and suitable execution of mathematics instruction in a bilingual child's home language as well as the dominant school language is only part of the whole picture. Student-level factors such as their pre-existing knowledge, cultural background, interests, and motivations must be given equal consideration to tailor effective mathematics instruction for bilingual students. Additionally, offloading bilingual students' persistent cognitive load by recruiting their other sensory modalities while creating an immersive learning environment can also help. Mixed methods research (Anchan & Soylu, 2021a-e) is currently underway to precisely target various aspects of this topic and address this educational concern pragmatically and cohesively.

References

Abedi, J., Lord, C., Hofstetter, C., & Baker, E. (2000). Impact of accommodation strategies on English language learners' test performance. *Educational Measurement: Issues and Practice*, *19*(3), 16-26.

Adesope, O. O., Lavin, T., Thompson, T., & Ungerleider, C. (2010). A systematic review and meta-analysis of the cognitive correlates of bilingualism. *Review of Educational Research*, *80*(2), 207–245.

Admiraal, W., Westhoff, G., & De Bot, K. (2006). Evaluation of bilingual secondary education in the Netherlands: Students' language proficiency in English 1. *Educational Research and Evaluation*, *12*(1), 75–93. https://doi.org/10.1080/13803610500392160

Aguirre-Muñoz, Z. & Pando, M. (2017). Knowing and teaching elementary math to bilingual students: examining the role of teaching self-efficacy on content knowledge. *Journal of Bilingual Education Research & Instruction, 19*(1)*,* 117–137.

Akiba, M., LeTendre, G. K., & Scribner, J. P. (2007). Teacher quality, opportunity gap, and national achievement in 46 countries. *Educational Researcher*, *36*(7), 369-387.

Anchan, D. M. (2019). *Beliefs about math learning and teaching among elementary school teachers and English learners* [Unpublished manuscript]. Department of Educational Studies in Psychology, Research Methods, and Counseling. The University of Alabama, Tuscaloosa, USA.

Anchan, D. M. (2022). *Incorporating Community-Based Participatory Research (CBPR) in a Research-Practice Cycle: The need for an updated epistemology in Educational Neuroscience* [Unpublished manuscript].

Department of Educational Studies in Psychology, Research Methods, and Counseling. The University of Alabama, Tuscaloosa, USA.

Anchan, D. M., & Soylu, F. (2021a). [Unpublished raw data on an *EEG time-frequency analysis of an arithmetic experiment comparing Spanish-English bilinguals and English monolinguals*]. The University of Alabama.

Anchan, D. M., & Soylu, F. (2021b). [Unpublished raw data on *phenomenological interviews with elementary-level math teachers about their experiences teaching bilingual children*]. The University of Alabama.

Anchan, D. M., & Soylu, F. (2021c). [Unpublished raw data on *phenomenological interviews with elementary-level Spanish-speaking English learners about their experiences learning math in their secondary language*]. The University of Alabama.

Anchan, D. M., & Soylu, F. (2021d). [Unpublished raw data on *multilevel modeling of TIMSS 2021 dataset to explore student-level, teacher-level, and school-level factors affecting mathematical performance*]. The University of Alabama.

Anchan, D. M., & Soylu, F. (2021e). [Unpublished raw data on *differential item functioning of TIMSS 2021 dataset to discover linguistic complexities in math test items*]. The University of Alabama.

Anderson, J. A., Chung-Fat-Yim, A., Bellana, B., Luk, G., & Bialystok, E. (2018). Language and cognitive control networks in bilinguals and monolinguals. *Neuropsychologia*, *117*, 352-363.

Anderson, J. R. (1983). A spreading activation theory of memory. *Journal of Verbal Learning and Verbal Behavior*, *22*(3), 261-295.

Anderson, M. (2010). Neural reuse: A fundamental organizational principle of the brain. *Behavioral and Brain Sciences*, *33*(4), 245.

Anderson, M. L. (2014). *After phrenology: Neural reuse and the interactive brain*. MIT Press.

Anstrom, K., & DiCerbo, P. (1998). *Preparing secondary education teachers to work with English language learners: English language arts. NCBE Resource Collection Series* (Report No. 10). National Clearinghouse for Bilingual Education.

Anstrom, K., DiCerbo, P., Butler, F., Katz, A., Millet, J., & Rivera, C. (2010). *A review of the literature on academic English: Implications for K-12 English language learners.* Arlington, VA: The George Washington University Center for Equity and Excellence in Education.

Arsalidou, M., & Taylor, M. J. (2011). Is 2+ 2= 4? Meta-analyses of brain areas needed for numbers and calculations. *Neuroimage*, *54*(3), 2382-2393.

Arsalidou, M., Pawliw-Levac, M., Sadeghi, M., & Pascual-Leone, J. (2018). Brain areas associated with numbers and calculations in children: Meta-analyses of fMRI studies. *Developmental Cognitive Neuroscience*, *30*, 239-250.

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, *44*(1-2), 75-106.

Baroody, A. J., & Wilkins, J. L. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48–65). National Association for the Education of Young Children.

Barth, H., Kanwisher, N., & Spelke, E. (2003). The construction of large number representations in adults. *Cognition*, *86*(3), 201-221.

Bedore, L. M., Peña, E. D., Griffin, Z. M., & Hixon, J. G. (2016). Effects of age of English exposure, current input/output, and grade on bilingual language performance. *Journal of Child Language*, *43*(3), 687-706.

Bender, A., & Beller, S. (2012). Nature and culture of finger counting: Diversity and representational effects of an embodied cognitive tool. *Cognition*, *124*(2), 156-182.

Benson, C. J. (2000). The primary bilingual education experiment in mozambique, 1993 to 1997. *International Journal of Bilingual Education and Bilingualism*, *3*(3), 149–166. https://doi.org/10.1080/13670050008667704

Bernardo, A. B. (2002). Language and mathematical problem solving among bilinguals. *The Journal of Psychology*, *136*(3), 283-297.

Bernardo, A. B. I., & Calleja, M. O. (2005). The effects of stating problems in bilingual students' first and second languages on solving mathematical word problems. *Journal of Genetic Psychology*, *166*(1), 117–129. https://doi.org/10.3200/GNTP.166.1.117-129

Berteletti, I., & Booth, J. R. (2015). Perceiving fingers in single-digit arithmetic problems. *Frontiers in Psychology*, *6*:*226*, 1–15.

Bialystok, E. (1999). Cognitive complexity and attentional control in the bilingual mind. *Child Development*, *70*(3), 636-644.

Bialystok, E., Craik, F. I., & Freedman, M. (2007). Bilingualism as a protection against the onset of symptoms of dementia. *Neuropsychologia*, *45*(2), 459-464.

Bialystok, E., Craik, F. I., & Luk, G. (2012). Bilingualism: Consequences for mind and brain. *Trends in Cognitive Sciences*, *16*(4), 240-250.

Bialystok, E., Craik, F., & Luk, G. (2008). Cognitive control and lexical access in younger and older bilinguals. *Journal of Experimental Psychology: Learning, memory, and cognition*, *34*(4), 859-873.

Bialystok, E., & Majumder, S. (1998). The relationship between bilingualism and the development of cognitive processes in problem solving. *Applied Psycholinguistics*, *19*(1), 69–85. https://doi.org/10.1017/s0142716400010584

Bialystok, E., & Martin, M. M. (2004). Attention and inhibition in bilingual children: Evidence from the dimensional change card sort task. *Developmental Science*, *7*(3), 325-339.

Blackburn, A. M. (2013). *A study of the relationship between code switching and the bilingual advantage: Evidence that language use modulates neural indices of language processing and cognitive control*. (Publication No. 1493850293) [Doctoral dissertation, The University of Texas at San Antonio]. ProQuest Dissertations & Theses Open. https://www.proquest.com/docview/1493850293

Bloechle, J., Huber, S., Bahnmueller, J., Rennig, J., Willmes, K., Cavdaroglu, S., ... & Klein, E. (2016). Fact learning in complex arithmetic—the role of the angular gyrus revisited. *Human Brain Mapping*, *37*(9), 3061-3079.

Bose, E., & Remillard, J. (2011). *Looking for equity in policy recommendations for instructional quality*. *Mapping Equity and Quality in Mathematics Education*. https://doi.org/10.1007/978-90-481-9803-0_13

Bryant, D. P. (2005). Commentary on early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*(4), 340-345.

Burton, F. D. (2011). *Fire: The spark that ignited human evolution*. UNM Press.

Butler, F. A., & Stevens, R. (1997). *Accommodation strategies for English language learners on large-scale assessments: Student characteristics and other considerations* (Vol. 448). Center for Research on Evaluation, National Center for Research on Evaluation, Standards, and Student Testing, Graduate School of Education & Information Studies, University of California, Los Angeles.

Butterworth, B. (1999). *The mathematical brain*. Macmillan Publishers.

Byers-Heinlein, K., Esposito, A. G., Winsler, A., Marian, V., Castro, D. C., & Luk, G. (2019). The case for measuring and reporting bilingualism in developmental research. *Collabra. Psychology*, *5*(1):37, 1-11.

Bynner, J., & Parsons, S. (1997). *Does numeracy matter? Evidence from the national child development study on the impact of poor numeracy on adult life*. Basic Skills Agency.

Cahnmann, M. S., & Remillard, J. T. (2002). What counts and how: Mathematics teaching in culturally, linguistically, and socioeconomically diverse urban settings. *Urban Review*, *34*(3), 179–204. https://doi.org/10.1023/A:1020619922685

Campbell, J. I. D., & Clark, J. M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, and Goodman (1986). *Journal of Experimental Psychology: General, 117*(2), 204–214.

Campbell, J. I. D., & Epp, L. J. (2004). An encoding-complex approach to numerical cognition in Chinese-English bilinguals. *Canadian Journal of Experimental Psychology*, *58* (4), 229–244.

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, *130*(2), 299.

Cardelle-Elawar, M. (1990). Effects of feedback tailored to bilingual students' mathematics needs on verbal problem solving. *The Elementary School Journal*, *91*(2), 165–175. https://doi.org/10.1086/461644

Cerda, V. R., Grenier, A. E., & Wicha, N. Y. Y. (2019). Bilingual children access multiplication facts from semantic memory equivalently across languages: Evidence from the N400. *Brain and Language*, *198*(August), 104679. https://doi.org/10.1016/j.bandl.2019.104679

Clarkson, P. C. (1992). Language and mathematics: A comparison of bilingual and monolingual students of mathematics. *Educational Studies in Mathematics,* 23(4), 417-429.

Clarkson, P. C., & Galbraith, P. (1992). Bilingualism and mathematics learning: Another perspective. *Journal for Research in Mathematics Education*, *23*(1), 34-44.

Cobb, P. (1987). An investigation of young children's academic arithmetic contexts. *Educational Studies in Mathematics*, *18*(2), 109-124.

Cohen, A. D. (1976). *The redwood city bilingual education project, 1971-1974: Spanish and English proficiency, mathematics, and language use over time.* Working Papers on Bilingualism, (Report No. 8). The Ontario Institute for Studies in Education.

Corballis, M. C. (2010). Mirror neurons and the evolution of language. *Brain and Language*, *112*(1), 25-35.

Costa, A., Hernández, M., & Sebastián-Gallés, N. (2008). Bilingualism aids conflict resolution: Evidence from the ANT task. *Cognition*, *106*(1), 59–86.

Crollen, V., & Noël, M. P. (2015). The role of fingers in the development of counting and arithmetic skills. *Acta Psychologica*, *156*, 37-44.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1-2), 1-42.

Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. OUP USA.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1*(1), 83-120.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*(3-6), 487-506.

Dick, A. S., Garcia, N. L., Pruden, S. M., Thompson, W. K., Hawes, S. W., Sutherland, M. T., ... & Gonzalez, R. (2019). No evidence for a bilingual executive function advantage in the ABCD study. *Nature Human Behaviour*, *3*(7), 692-701.

Dijkstra, T., & Van Heuven, W. (2002). The architecture of the bilingual word recognition system: From identification to decision. *Bilingualism: Language and Cognition, 5*(3), 175-197.

Domínguez, H. (2011). Using what matters to students in bilingual mathematics problems. *Educational Studies in Mathematics*, *76*(3), 305–328. https://doi.org/10.1007/s10649-010-9284-z

Dowker, A. (2019). *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. Routledge.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428.

Faye, A., Jacquin-Courtois, S., Reynaud, E., Lesourd, M., Besnard, J., & Osiurak, F. (2019). Numerical cognition: A meta-analysis of neuroimaging, transcranial magnetic stimulation and brain-damaged patients studies. *NeuroImage: Clinical*, *24*, 102053.

Fayol, M., Barrouillet, P., & Marinthe, C. (1998). Predicting arithmetical achievement from neuro-psychological performance: A longitudinal study. *Cognition*, *68*(2), B63-B70.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307-314.

Fias, W., Lammertyn, J., Reynvoet, B., Dupont, P., & Orban, G. A. (2003). Parietal representation of symbolic and nonsymbolic magnitude. *Journal of Cognitive Neuroscience*, *15*(1), 47-56.

Fischer, U., Suggate, S. P., Schmirl, J., & Stoeger, H. (2018). Counting on fine motor skills: links between preschool finger dexterity and numerical skills. *Developmental Science*, *21*(4), e12623.

Friesen, D. C., Luo, L., Luk, G., & Bialystok, E. (2015). Proficiency and control in verbal fluency performance across the lifespan for monolinguals and bilinguals. *Language, Cognition and Neuroscience*, *30*(3), 238-250.

Fry, R. (2007). *How far behind in math and reading are English language learners?* (Report). Washington, DC: Pew Hispanic Center.

Fry, R. (2008). *The role of schools in the English language learner achievement gap* (Report). Washington, DC: Pew Hispanic Center.

Gale, K., McClay, D., Christie, M., & Harris, S. (1981). Academic achievement in the milingimbi bilingual education program. *TESOL Quarterly*, *15*(3), 297. https://doi.org/10.2307/3586755

Gallagher, K. (2011). Bilingual education in the UAE: Factors, variables and critical questions. *Education, Business and Society: Contemporary Middle Eastern Issues*, *4*(1), 62–79. https://doi.org/10.1108/17537981111111274

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, *37*(1), 4-15.

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: a 5-year longitudinal study. *Developmental Psychology*, *47*(6), 1539–1552.

Geary, D. C., Berch, D. B., & Koepke, K. M. (2014). *Evolutionary origins and early development of number processing*. Academic Press.

Geary, D. C., Bow-Thomas, C. C., Fan, L., & Siegler, R. S. (1993). Even before formal instruction, Chinese children outperform American children in mental addition. *Cognitive Development*, *8*(4), 517-529.

Geary, D. C., Bow-Thomas, C. C., Liu, F., & Siegler, R. S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development*, *67*(5), 2022-2044.

Goldstein, T. (2003). Contemporary bilingual life at a Canadian high school: Choices, risks, tensions, and dilemmas. *Sociology of Education*, *76*(3), 247–264. https://doi.org/10.2307/3108468

Goodrich, J. M., Lonigan, C. J., & Farver, J. M. (2013). Do early literacy skills in children's first language promote development of skills in their second language? An experimental evaluation of transfer. *Journal of Educational Psychology*, *105*(2), 414–426.

Grabner, R. H., Ansari, D., Koschutnig, K., Reishofer, G., Ebner, F., & Neuper, C. (2009). To retrieve or to calculate? Left angular gyrus mediates the retrieval of arithmetic facts during problem solving. *Neuropsychologia*, *47*(2), 604-608.

Grabner, R. H., Ansari, D., Reishofer, G., Stern, E., Ebner, F., & Neuper, C. (2007). Individual differences in mathematical competence predict parietal brain activation during mental calculation. *Neuroimage*, *38*(2), 346-356.

Greene, J. P. (1997). A meta-analysis of the Rossell and Baker review of bilingual education research. *Bilingual Research Journal*, *21*(2–3), 103–122. https://doi.org/10.1080/15235882.1997.10668656

Grosjean, F. (1989). Neurolinguists, beware! The bilingual is not two monolinguals in one person. *Brain and language*, *36*(1), 3-15.

Gutek, G. L. (1994). *A history of the western educational experience*. Waveland Press.

Gutiérrez, R. (2002). Beyond essentialism: The complexity of language in teaching mathematics to Latina/o students. *American Educational Research Journal*, *39*(4), 1047–1088. https://doi.org/10.3102/000283120390041047

Han, H., Soylu, F., & Anchan, D. M. (2019). Connecting levels of analysis in educational neuroscience: a review of multi-level structure of educational neuroscience with concrete examples. *Trends in Neuroscience and Education*, *17*, 100113.

Institute of Education Sciences – National Center for Education Statistics (2020). *English language learners in public schools*. https://nces.ed.gov/programs/coe/indicator_cgf.asp#

Jankowiak, K., & Rataj, K. (2017). The N400 as a window into lexico-semantic processing in bilingualism. *Poznan Studies in Contemporary Linguistics*, *53*(1), 119–156. https://doi.org/10.1515/psicl-2017-0006

Jitendra, A. K., Lein, A. E., Star, J. R., & Dupuis, D. N. (2013). The contribution of domain-specific knowledge in predicting students' proportional word problem-solving performance. *Educational Research and Evaluation*, *19*(8), 700–716. https://doi.org/10.1080/13803611.2013.845107

Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second-grade children with different forms of LD. *Journal of Learning Disabilities*, *33*(6), 567-578.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2008). Development of number combination skill in the early school years: When do fingers help? *Developmental Science*, *11*(5), 662-668.

Kempert, S., Saalbach, H., & Hardy, I. (2011). Cognitive benefits and costs of bilingualism in elementary school students: The case of mathematical word problems. *Journal of Educational Psychology*, *103*(3), 547–561. https://doi.org/10.1037/a0023619

King, Y. A., & Purpura, D. J. (2021). Direct numeracy activities and early math skills: Math language as a mediator. *Early Childhood Research Quar-*

terly, *54*, 252-259.Knox, R. (2016). Mind, brain, and education: A transdisciplinary field. *Mind, Brain, and Education*, *10*(1), 4-9.

Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and instruction*, *25*, 95-103.

Kroll, J. F. (2008). Juggling two languages in one mind. *Psychological Science Agenda, 22*(1), 1-6. https://www.apa.org/science/about/psa/2008/01/kroll

Kroll, J. F., Bobb, S. C., Misra, M., & Guo, T. (2008). Language selection in bilingual speech: Evidence for inhibitory processes. *Acta Psychologica*, *128*(3), 416-430.

Kroll, J. F., Dussias, P. E., Bice, K., & Perrotti, L. (2015). Bilingualism, mind, and brain. *Annual Review of Linguistics*, *1*(1), 377-394.

Lambert, W. E., Genesee, F., Holobow, N., & Chartrand, L. (1993). Bilingual education for majority English-speaking children. *European Journal of Psychology of Education*, *8*(1), 3–22. https://doi.org/10.1007/BF03172860

Lau, S. M. C. (2013). A study of critical literacy work with beginning English language learners: An integrated approach. *Critical Inquiry in Language Studies*, *10*(1), 1-30.

Lee, S., Watt, R., & Frawley, J. (2015). Effectiveness of bilingual education in Cambodia: A longitudinal comparative case study of ethnic minority children in bilingual and monolingual schools. *Compare*, *45*(4), 526–544. https://doi.org/10.1080/03057925.2014.909717

Lim, C. S., & Chew, C. M. (2007, December 9-14). *Mathematical Communication in Malaysian Bilingual Classrooms* [Paper presentation]. APEC-Tsukuba International Conference 2007: Innovation of classroom teaching and learning through lesson study-focusing on mathematical communication, Tokyo, Japan. Retrieved from http://www.criced.tsukuba.ac.jp/math/apec/apec2008/papers/PDF/11.LimChapSam_Malaysia.pdf

Lindholm-Leary, K., & Borsato, G. (2005). Hispanic high schoolers and mathematics: Follow-up of students who had participated in two-way bilingual elementary programs. *Bilingual Research Journal*, *29*(3), 641–652. https://doi.org/10.1080/15235882.2005.10162856

Long, I., Malone, S. A., Tolan, A., Burgoyne, K., Heron-Delaney, M., Witteveen, K., & Hulme, C. (2016). The cognitive foundations of early arithmetic skills: It is counting and number judgment, but not finger gnosis, that count. *Journal of Experimental Child Psychology*, *152*, 327-334.

Lubinski, D., Benbow, C. P., & Kell, H. J. (2014). Life paths and accomplishments of mathematically precocious males and females four decades later. *Psychological Science*, *25*(12), 2217-2232.

Luk, G., & Bialystok, E. (2013). Bilingualism is not a categorical variable: Interaction between language proficiency and usage. *Journal of Cognitive Psychology*, 25 (5), 605–621.

Luo, Z., Jose, P. E., Huntsinger, C. S., & Pigott, T. D. (2007). Fine motor skills and mathematics achievement in East Asian American and European

American kindergartners and first graders. *British Journal of Developmental Psychology*, *25*(4), 595-614.

Marian, V., Shook, A., & Schroeder, S. R. (2013). Bilingual two-way immersion programs benefit academic achievement. *Bilingual Research Journal*, 36(2), 167-186.

Martinez, R. D. (1999). *Assessment: A development guidebook for teachers of English-language learners* (Report). Washington, DC: Office of Educational Research and Improvement.

Martinez-Lincoln, A., Cortinas, C., & Wicha, N. Y. Y. (2015). Arithmetic memory networks established in childhood are changed by experience in adulthood. *Neuroscience Letters*, *584*, 325–330. https://doi.org/10.1016/j.neulet.2014.11.010

Menon, V. (2015). *Arithmetic in the child and adult brain.* In R. C. Kadosh & A. Dowker (Eds.), *Oxford library of psychology. The Oxford handbook of numerical cognition* (pp. 502–530). Oxford University Press.

Mielicki, M. K., Kacinik, N. A., & Wiley, J. (2017). Bilingualism and symbolic abstraction: Implications for algebra learning. *Learning and Instruction*, *49*, 242–250.

Miller, K. F., & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive Development*, *2*(3), 279-305.

Moschkovich, J. (2007a). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nassir. and Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 89–104). Teachers College Press.

Moschkovich, J. (2007b). Using two languages when learning mathematics. *Educational Studies in Mathematics*, *64*(2), 121–144. https://doi.org/10.1007/s10649-005-9005-1

Musanti, S. I., Celedón-Pattichis, S., & Marshall, M. E. (2009). Reflections on language and mathematics problem solving: A case study of a bilingual first-grade teacher. *Bilingual Research Journal*, *32*(1), 25–41. https://doi.org/10.1080/15235880902965763

National Assessment of Educational Progress (2019). *2019 Mathematics Assessment.* https://www.nationsreportcard.gov/mathematics/supportive_files/2019_Results_Appendix_Math_State.pdf

National Center for Education Statistics (2017). *The nation's report card: A first look: 2017 mathematics and reading* (NCES 2017–451). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education, Washington, D.C.

Newman, S. D., & Soylu, F. (2014). The impact of finger counting habits on arithmetic in adults and children. *Psychological Research*, *78*(4), 549-556.

Ngan Ng, S. S., & Rao, N. (2010). Chinese number words, culture, and mathematics learning. *Review of Educational Research*, *80*(2), 180-206.

Nichols, E. S., Wild, C. J., Stojanoski, B., Battista, M. E., & Owen, A. M. (2020). Bilingualism affords no general cognitive advantages: A population study of executive function in 11,000 People. *Psychological Science*, *31*(5), 548-567.

Noël, M. P. (2005). Finger gnosia: a predictor of numerical abilities in children? *Child Neuropsychology*, *11*(5), 413-430.

Norén, E. (2008). Bilingual students mother tongue: a resource for teaching and learning mathematics. *Nordic Studies in Mathematics Education*, *13*(4), 29–50.

O'Shaughnessy, D. M., Gibson, E., & Piantadosi, S. T. (2021). The cultural origins of symbolic number. *Psychological Review.* Advance online publication. https://doi.org/10.1037/rev0000289

Özerk, K. Z. (1996). Linguistic-minority students and bilingual mathematics teaching. *Scandinavian Journal of Educational Research*, 40(4), 333-348.

Park, D., Gunderson, E. A., Tsukayama, E., Levine, S. C., & Beilock, S. L. (2016). Young children's motivational frameworks and math achievement: Relation to teacher-reported instructional practices, but not teacher theory of intelligence. *Journal of Educational Psychology*, *108*(3), 300.

Parvanehnezhad, Z., & Clarkson, P. C. (2008). Iranian bilingual students reported use of language switching when doing mathematics. *Mathematics Education Research Journal*, *20*(1), 52–81. https://doi.org/10.1007/BF03217469

Patrinos, H. A., & Velez, E. (2009). Costs and benefits of bilingual education in Guatemala: A partial analysis. *International Journal of Educational Development*, *29*(6), 594–598. https://doi.org/10.1016/j.ijedudev.2009.02.001

Peterson, M. W., & Rivera, C. (1994). Is it real for all kids? A framework for equitable assessment policies for English language learners. *Harvard Educational Review*, *64*(1), 55.

Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: Tensions in the bilingual mathematics classroom. *Mathematics Education Research Journal*, *25*(3), 361–378. https://doi.org/10.1007/s13394-013-0075-6

Prado, J., Mutreja, R., Zhang, H., Mehta, R., Desroches, A. S., Minas, J. E., & Booth, J. R. (2011). Distinct representations of subtraction and multiplication in the neural systems for numerosity and language. *Human Brain Mapping*, *32*(11), 1932-1947.

Qohar, A., & Sumarmo, U. (2013). Improving mathematical communication ability and self-regulation learning of junior high students by using reciprocal teaching. *Indonesian Mathematical Society Journal on Mathematics Education*, 4(1), 59–74.

Roberts, G., & Bryant, D. (2011). Early mathematics achievement trajectories: English-language learner and native English-speaker estimates, using the early childhood longitudinal survey. *Developmental Psychology*, *47*(4), 916.

Rosenberg-Lee, M., Barth, M., & Menon, V. (2011). What difference does a year of schooling make? Maturation of brain response and connectivity between 2nd and 3rd grades during arithmetic problem solving. *Neuroimage*, *57*(3), 796-808.

Rossell, C. H., & Baker, K. (1996). The educational effectiveness of bilingual education. *Research in the Teaching of English*, *30*(1), 7-74.

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, *102*(3), 361-395.

Ruiz-Felter, R., Cooperson, S. J., Bedore, L. M., & Peña, E. D. (2016). Influence of current input–output and age of first exposure on phonological acquisition in early bilingual Spanish–English-speaking kindergarteners. *International Journal of Language & Communication Disorders*, *51*(4), 368-383.

Saalbach, H., Eckstein, D., Andri, N., Hobi, R., & Grabner, R. H. (2013). When language of instruction and language of application differ: Cognitive costs of bilingual mathematics learning. *Learning and Instruction*, *26*, 36-44.

Salillas, E., & Wicha, N. Y. Y. (2012). Early learning shapes the memory networks for arithmetic: Evidence from brain potentials in bilinguals. *Psychological Science*, *23*(7), 745–755. https://doi.org/10.1177/0956797612446347

Saxe, G. B., & Esmonde, I. (2012). *Cultural development of mathematical ideas: Papua New Guinea studies*. Cambridge University Press.

Schwartz, B. D., & Sprouse, R. A. (1996). L2 cognitive states and the full transfer/full access model. *Second Language Research*, *12*(1), 40-72.

Secada, W. G., & De La Cruz, Y. (1996). *Teaching mathematics for understanding to bilingual students* (Report). Madison, WI: National Center for Research In Mathematical Sciences Education.

Sedig, K., & Liang, H. N. (2006). Interactivity of visual mathematical representations: Factors affecting learning and cognitive processes. *Journal of Interactive Learning Research*, *17*(2), 179-212.

Selin, H. (Ed.). (2001). *Mathematics across cultures: The history of non-western mathematics*. Kluwer Academic Publishers.

Shin, S. J. (2013). *Bilingualism in schools and society: Language, identity, and policy*. Routledge.

Siegler, R. S., & Braithwaite, D. W. (2017). Numerical development. *Annual Review of Psychology*, *68*, 187-213.

Soylu, F., & Newman, S. D. (2016). Anatomically ordered tapping interferes more with one-digit addition than two-digit addition: A dual-task fMRI study. *Cognitive Processing*, *17*(1), 67-77.

Soylu, F., Lester Jr, F. K., & Newman, S. D. (2018). You can count on your fingers: The role of fingers in early mathematical development. *Journal of Numerical Cognition*, *4*(1), 107-135.

Soylu, F., Raymond, D., Gutierrez, A., & Newman, S. D. (2018). The differential relationship between finger gnosis, and addition and subtraction: An fMRI study. *Journal of Numerical Cognition*, *3*(3), 694-715.

Sterelny, K. (2012). Language, gesture, skill: The co-evolutionary foundations of language. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *367*(1599), 2141-2151.

Stern, Y. (2009). Cognitive reserve. *Neuropsychologia*, *47*(10), 2015-2028.

Surrain, S., & Luk, G. (2019). Describing bilinguals: A systematic review of labels and descriptions used in the literature between 2005–2015. *Bilingualism: Language and Cognition*, *22*(2), 401-415.

Tang, Y., Zhang, W., Chen, K., Feng, S., Ji, Y., Shen, J., ... & Liu, Y. (2006). Arithmetic processing in the brain shaped by cultures. *Proceedings of the National Academy of Sciences*, *103*(28), 10775-10780.

Thordardottir, E. (2019). Amount trumps timing in bilingual vocabulary acquisition: Effects of input in simultaneous and sequential school-age bilinguals. *International Journal of Bilingualism*, *23*(1), 236-255.

Tong, F., Lara-Alecio, R., Irby, B. J., & Mathes, P. G. (2011). The effects of an instructional intervention on dual language development among first-grade

Hispanic English-learning boys and girls: A two-year longitudinal study. *Journal of Educational Research*, *104*(2), 87-99.

Troike, R. C. (1978). Research evidence for the effectiveness of bilingual education. *NABE Journal*, *3*(1), 13–24. https://doi.org/10.1080/08855072.1978.10668341

Tsung, L. T. H., & Cruickshank, K. (2009). Mother tongue and bilingual minority education in China. *International Journal of Bilingual Education and Bilingualism*, *12*(5), 549–563. https://doi.org/10.1080/13670050802209871

U.S. Census Bureau (2021). Language spoken at home – States 2015-2019 (Version April 12, 2021) [Data Table]. https://covid19.census.gov/datasets/language-spoken-at-home-states-2015-2019/explore.

U.S. Department of Education, National Center for Education Statistics, Common Core of Data (CCD), "Local education agency universe survey," 2018–19. See *Digest of Education Statistics 2020*, table 204.20 (2020). https://nces.ed.gov/programs/coe/indicator/cgf#:%7E:text=The%20percentage%20of%20public%20school,%2C%20or%203.8%20million%20students

U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2019 Mathematics Assessment (2021). https://www.nationsreportcard.gov/ndecore/xplore/NDE

United States Department of Education (2019). *English learners and high school mathematics*. https://ncela.ed.gov/files/fast_facts/FactSheet_Del4.4_HighSchoolMath_112619_508.pdf

van Heuven, W. J., & Dijkstra, T. (2010). Language comprehension in the bilingual brain: fMRI and ERP support for psycholinguistic models. *Brain Research Reviews*, *64*(1), 104-122.

Van Rinsveld, A., Brunner, M., Landerl, K., Schiltz, C., & Ugen, S. (2015). The relation between language and arithmetic in bilinguals: insights from different stages of language acquisition. *Frontiers in Psychology*, *6*, 265.

Van Rinsveld, A., Schiltz, C., Brunner, M., Landerl, K., & Ugen, S. (2016). Solving arithmetic problems in first and second language: Does the language context matter? *Learning and Instruction*, *42*, 72-82.

Venkatraman, V., Soon, C. S., Chee, M. W. L., & Ansari, D. (2006). Effect of language switching on arithmetic: A bilingual fMRI study. *Journal of Cognitive Neuroscience*, *18*(1), 64–74. https://doi.org/10.1162/089892906775250030

WestEd (2020, August). *Perspectives on English Language Learning: Aida Walqui in Conversation with Leading Scholars, Quality Teaching for English Learners at WestEd.* [Webinar Series]. Retrieved from https://www.wested.org/wested_event/qtel-conversation-series-2020/

White, L. (2003). *Second language acquisition and universal grammar*. Cam bridge University Press.

Whitford, V. & Luk, G. (2019). Comparing executive functions in monolinguals and bilinguals. In *Bilingualism, executive function, and beyond: Questions and insights* (Vol. 57, pp. 67-80). John Benjamins Publishing Company.

Wicha, N. Y., Dickson, D. S., & Martinez-Lincoln, A. (2018). Arithmetic in the bilingual brain. In *Language and culture in mathematical cognition* (pp. 145-172). Academic Press. https://doi.org/10.1016/b978-0-12-812574-8.00007-9

Willig, A. C. (2012). A meta-analysis of selected studies on the effectiveness of bilingual education. *The New Immigration: An Interdisciplinary Reader*, *55*(3), 249–287. https://doi.org/10.4324/9780203621028-22

Wolf, M. K., & Leon, S. (2009). An investigation of the language demands in content assessments for English language learners. *Educational Assessment*, *14*(3-4), 139-159.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*(1), B1-B11.

Yeh, C. (2017). Math Is more than numbers: Beginning bilingual teachers' mathematics teaching practices and their opportunities to learn. *Journal of Urban Mathematics Education*, *10*(2), 106–139.

Zago, L., Pesenti, M., Mellet, E., Crivello, F., Mazoyer, B., & Tzourio-Mazoyer, N. (2001). Neural correlates of simple and complex mental calculation. *Neuroimage*, *13*(2), 314-327.

Zaretskii, V. K. (2009). The zone of proximal development: What Vygotsky did not have time to write. *Journal of Russian & East European Psychology*, *47*(6), 70-93.